**Real-Time Co-optimization Model**

**Objective Function (probabilistic model):**

Max ∑d [B(d-u) - C(g)] f(d) + DRUS(dRUS) + DRRS(dRRS) - CSOR(rSOR) - CRUS(rRUS) - CRRS(rRRS)

Or, simplifying by setting B(d) = VOLL\*d, making d and g the expected values of net load (GTBD) and generation, and x with mean 0 representing the uncertainty in net load:

Max ∑x [VOLL(d+x-u) - C(g+g')] f(x) + DRUS(dRUS) + DRRS(dRRS) - CSOR(rSOR) - CRUS(rRUS) - CRRS(rRRS)

**Subject to:**

Ignoring transmission constraints and focusing on power balance, AS procurement and Resource limit constraints, the set of constraints are given below:

**System wide constraints:**

1. Expected Power Balance: (Shadow price = $λ$) g - d = 0
2. RegUp Procurement: (Shadow price = $α$) rRUS - dRUS ≥ 0
3. RRS Procurement: (Shadow price = $γ$) rRRS - dRRS ≥ 0

**Individual Resource constraints:**

Each Resource will have its own set of constraints to ensure awards are within bounds of its own upper (HSL/MPC) and low (LSL/LPC) limits.

1. LSL Constraint: (Shadow price = $θ\_{i}^{LSL}$) g - LSL ≥ 0
2. HSL Constraint: (Shadow price = $θ\_{i}^{HSL}$) HSL - g - rRUS - rRRS - rSOR ≥ 0
3. SOR capacity constraint: (Shadow price = $θ\_{x}^{SOR}$) rSOR - g' ≥ 0
4. SOR dispatch constraint: (Shadow price = $λ\_{x}^{'}$) x -u - g' = 0

**Lagrangian Function:**

The objective and constraints are combined to form the Lagrange function:

$$L=\left\{Objective- \sum\_{i}^{}Shadowprice\_{i}×Constraint\_{i}\right\}$$

At optimal solution $∇L=0$ (optimality condition)

i.e. the partial derivative of $L$ with respect to each award g, rRUS, rRRS, rSOR, and the shadow prices $\left(λ,α,γ,λ', θ\_{i}^{SOR},θ\_{i}^{LSL},θ\_{i}^{HSL}, etc.\right)$ will equate to zero at the optimal solution.

 Taking the partial derivative of $L$ with respect to each award and rearranging the terms,$ $we get:

1. For the expected energy award, g:

$$\sum\_{x}^{}C^{'}(g+g') f(x)=λ-θ\_{i}^{HSL}+θ\_{i}^{LSL} $$

If the energy offer *i* is marginal to the power balance constraint, then, $θ\_{i}^{HSL}=0,θ\_{i}^{LSL}=0 $ and the energy offer *i* sets the shadow price for the power balance constraint (System Lambda $\left(λ\right)$).

1. For the energy dispatch from SOR capacity for each x, g':

$$C'(g+g') =λ\_{x}^{'}-θ\_{x}^{SOR} $$

If the energy dispatch *i* is marginal to the power balance constraint, then, $θ\_{i}^{SOR}=0 $ and the energy offer *i* sets the shadow price for the SOR dispatch constraint.

1. For the RegUp award, rRUS:

$$C\_{RUS}^{'}= α-θ\_{i}^{HSL}$$

If the RegUp offer *i* is marginal to the RegUp Procurement constraint, then in most cases, $θ\_{i}^{HSL}=0, $and the RegUp Offer *i* sets the shadow price for the RegUp Procurement constraint (RegUp MCPC $\left(α\right)$ )

1. For the RRS award, rRRS:

$$C\_{RRS}^{'}= γ-θ\_{i}^{HSL}$$

If the RRS offer *i* is marginal to the RRS Procurement constraint, then, $θ\_{i}^{HSL} $ and the RRS Offer *i* sets the shadow price for the RRS Procurement constraint (RRS MCPC $\left(γ\right)$ )

1. For the SOR award, rSOR:

$$C\_{SOR}^{'}= \sum\_{x}^{}θ\_{x}^{SOR}f(x)-θ\_{i}^{HSL}$$

Or, substituting $θ\_{x}^{SOR}$ from above:

$$C\_{SOR}^{'}+ \sum\_{x}^{}C^{'}(g+g') f(x)=\sum\_{x}^{}λ\_{x}^{'}f(x)-θ\_{i}^{HSL}$$

If the SOR offer *i* is marginal to the SOR Procurement constraint, then, $θ\_{i}^{HSL}=0$ and the marginal cost of the capacity offer plus the expected marginal cost of energy offer equals the expected shadow price of SOR dispatch constraint.

**MCPC**

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| AS Product | MCPC | Comments |
| RegUp  | $$α$$ | Shadow price of the RegUp procurement constraint |
| RegDn  | $$β$$ | Shadow price of the RegDn procurement constraint |
| RRS | $$γ$$ | Shadow Price of the RRS procurement constraint |
| SOR | $$\sum\_{x}^{}λ\_{x}^{'}f(x)$$ | Expected value of the Shadow Prices of the SOR capacity constraints |