**Applying Karush-Kuhn-Tucker (KKT) Optimality Conditions for Pricing Analysis in DAM under the proposed Future AS**

**Introduction to concepts**

The simplified hypothetical DAM optimization problem is presented below.

Simplifications:

1. Transmission constraints, PTPs, block offers and bids are not considered
2. A single AS product (generically called AS) is considered
3. Load Resources are not considered
4. Energy Offer and AS Offer are covering the entire MW range from LSL to HSL
5. No constraints on how much AS can be awarded to a single resource
6. are the submitted bids ($/MWh) , energy offers ($/MWh), AS offers ($/MW) respectively. For simplicity, these bids and offers are considered to be constant for the entire MW bid or offered.
7. Resource commitment not considered – all Resources considered to be online and only online AS and Energy Offers considered
8. All AS Offers from Generation Resources are considered to be inclusive with respect to their EOC.
9. Temporal constraints not modeled
10. AS Self Arrangement set to zero
11. Other simplifications compared to actual DAM ….

**Optimization Problem**

**Minimize objective (cost) function:**

**Subject to:**

Ignoring transmission constraints and focusing on power balance, AS procurement and Resource limit constraints, the set of constraints are given below:

**System wide constraints:**

1. Power Balance: (Shadow price = )
2. AS Procurement: (Shadow price = )

**Individual Energy Bid constraints:**

1. Energy Bid MW constraint for every energy bid : (Shadow price = respectively)

**Individual Resource constraints:**

Each Resource will have its own set of constraints to ensure awards are within bounds of its own upper (HSL/MPC) and low (LSL/LPC) limits.

1. LSL Constraint for every modeled Generation Resource : (Shadow price = )
2. HSL Constraint for every modeled Generation Resource : (Shadow price = )

**Analysis by Applying KKT optimality conditions:**

The objective and constraints are combined to form the Lagrange function:

At optimal solution (optimality condition)

i.e. the partial derivative of with respect to each award will equate to zero at the optimal solution.

Taking the partial derivative of with respect to each award :

Rearranging the terms by we get:

1. For each energy bid , the following equation holds true

If the energy bid *i* is marginal to the power balance constraint, then and the energy bid *i* sets the shadow price for the power balance constraint (System Lambda )

1. For each Resource energy offer , the following equation holds true

If the energy offer *i* is marginal to the power balance constraint, then in most cases, and the energy offer *i* sets the shadow price for the power balance constraint (System Lambda )

1. For each Resource AS offer , the following equation holds true

If the AS offer *i* is marginal to the AS procurement constraint, then in most cases, and the AS offer *i* sets the shadow price for the AS procurement constraint (typically referred to as the AS MCPC )

**Pricing Analysis Example:**

Let there be a Resource g whose capacity has been partly awarded energy and partly awarded AS such that the sum of the energy award and AS award equals Resource g HSL.

In this case:

The HSL constraint is binding, i.e.

The LSL constraint is not binding, i.e.

And,

Rearranging we get

Equating the two equations we get

This demonstrates that at the optimal solution Resource g is indifferent to whether its capacity is used for energy or AS. This is because for each MW sold from the resource for energy or AS, the additional revenue from energy or AS, over submitted offer price, is the same.

**DAM Pricing Analysis Applying KKT Optimality Conditions– Future AS**

The simplified DAM optimization problem under the proposed Future Ancillary Service framework is presented below.

Simplifications:

1. Transmission constraints, PTPs, block offers and bids are not considered
2. Energy Offer are covering the entire MW range from LSL to HSL
3. No constraints on how much AS can be awarded to a single resource but AS awards cannot exceed the MW amount in the AS Offer
4. are the submitted bids ($/MWh) , energy offers ($/MWh), RegUp offers ($/MW), etc. respectively. For simplicity, these bids and offers are considered to be constant for the entire MW bid or offered.
5. Resource commitment not considered – all Resources considered to be online and only online AS and Energy Offers considered
6. All AS Offers from Generation Resources are considered to be inclusive with respect to their EOC.
7. Temporal constraints not modeled
8. AS Self Arrangement set to zero
9. Other simplifications compared to actual DAM ….

**Optimization Problem**

**Minimize objective (cost) function:**

**Subject to:**

Ignoring transmission constraints and focusing on power balance, AS procurement and Resource limit constraints, the set of constraints are given below:

**System wide constraints:**

1. Power Balance: (Shadow price = )
2. RegUp Procurement (including FRRS-Up): (Shadow price = )
3. FRRS-Up maximum procurement limit: (Shadow price = )
4. RegDn Procurement (including FRRS-Dn): (Shadow price = )
5. FRRS-Dn maximum procurement limit: (Shadow price = )
6. CR Procurement (CR1 & CR2): (Shadow price = )
7. CR1 minimum procurement : (Shadow price = )
8. SR Procurement (SR1 & SR2): (Shadow price = )
9. SR1 minimum procurement : (Shadow price = )
10. PFR+FFR1+FFR2 Procurement: (Shadow price = )
11. FFR1+FFR2 Maximum procurement limit : (Shadow price = )
12. FFR1 Maximum procurement limit : (Shadow price = )

**Individual Energy Bid constraints:**

1. Energy Bid MW constraint for every energy bid : (Shadow price = respectively)

**Individual Resource constraints:**

Each Resource will have its own set of constraints to ensure awards are within bounds of its own upper (HSL/MPC) and low (LSL/LPC) limits.

1. LSL Constraint for every modeled Generation Resource : (Shadow price = )
2. HSL Constraint for every modeled Generation Resource : (Shadow price = )
3. AS Offer MW constraint for every modeled Generation Resource : (Shadow price = respectively)
4. MPC & LPC Constraint for every modeled “Blocky” Load Resource : (Shadow price = )

Note that a “blocky” Load Resource is awarded only one AS product.

or

or

or

1. AS Offer MW constraint for every modeled “Blocky” Load Resource : (Shadow price = )

or

or

or

1. MPC & LPC Constraint for every modeled Controllable Load Resource : (Shadow price = )
2. AS Offer MW constraint for every Controllable Load Resource : (Shadow price = respectively)
3. HSL & LSL Constraint for every “Quick/Fast” Resource qualified for FRRS-Up and FFR1 and partly modeled as Generation Resource : (Shadow price = )
4. AS Offer MW constraint for every “Quick/Fast” Resource qualified for FRRS-Up and FFR1 and partly modeled as Generation Resource : (Shadow price = )
5. MPC & LPC Constraint for every “Quick/Fast” Resource qualified for FRRS-Up, FRRS-Dn and FFR1 and partly modeled as Controllable Load Resource : (Shadow price = )
6. AS Offer MW constraint for every “Quick/Fast” Resource qualified for FRRS-Up and FFR1 and partly modeled as Controllable Load Resource : (Shadow price = respectively)

**Pricing Analysis By Applying KKT optimality conditions:**

The objective and constraints are combined to form the Lagrange function:

At optimal solution (KKT optimality condition)

i.e. the partial derivative of with respect to each award will equate to zero at the optimal solution.

Taking the partial derivative of with respect to each award :

Rearranging the terms by we get:

1. For each energy bid , the following equation holds true
2. For each energy offer from modeled Generation Resource , the following equation holds true
3. For each RegUp offer from modeled Generation Resource , the following equation holds true
4. For each RegDn offer from modeled Generation Resource , the following equation holds true
5. For each PFR offer from modeled Generation Resource , the following equation holds true
6. For each CR1 offer from modeled Generation Resource , the following equation holds true
7. For each SR1 offer from modeled Generation Resource , the following equation holds true
8. For each FFR1 offer from modeled “blocky” Load Resource , the following equation holds true
9. For each FFR2 offer from modeled “blocky” Load Resource , the following equation holds true
10. For each CR2 offer from modeled “blocky” Load Resource , the following equation holds true
11. For each SR2 offer from modeled “blocky” Load Resource , the following equation holds true
12. For each RegUp offer from modeled Controllable Load Resource , the following equation holds true
13. For each RegDn offer from modeled Controllable Load Resource , the following equation holds true
14. For each PFR offer from modeled Controllable Load Resource , the following equation holds true
15. For each CR1 offer from modeled Controllable Load Resource , the following equation holds true
16. For each SR1 offer from modeled Controllable Load Resource , the following equation holds true
17. For each FRRS-Up offer from “Quick/Fast” Resource partly modeled as Generation Resource , the following equation holds true
18. For each FFR1 offer from “Quick/Fast” Resource partly modeled as Generation Resource , the following equation holds true
19. For each FRRS-Up offer from “Quick/Fast” Resource partly modeled as Load Resource , the following equation holds true
20. For each FRRS-Dn offer from “Quick/Fast” Resource partly modeled as Load Resource , the following equation holds true
21. For each FFR1 offer from “Quick/Fast” Resource partly modeled as Load Resource , the following equation holds true

**MCPC for proposed Future Ancillary Service**

MCPC for a given AS is computed after the DAM optimization is complete. MCPCs are a linear combination of Shadow Prices of constraints. In the current Nodal Market, the MCPCs are setup to be the Shadow Price of a single constraint (procurement constraint of the respective AS).

1. In the current ERCOT market, all demand (load) is bought at the Load Zone (except for Wholesale Load). Load Resources cannot submit Resource Specific (locational) demand bids (bid to buy energy). Load Resources can only submit Resource specific Offers to sell Ancillary Services.
2. Hence, the Load Resource Limit Constraints do not have an energy consumed portion between their lower and upper limits.
3. Generation Resources, however, can submit Resource specific Offers to sell energy and Ancillary Services.
4. Due to the above two features of our market design, in the DAM optimization:
   1. For Generation Resources, the energy and AS co-optimization process allocates the offered MW capacity (constrained between LSL and HSL) between energy and Ancillary Services taking into account opportunity costs for energy and Ancillary Services. i.e. the prices (LMP-energy and MCPC-AS) incorporate opportunity costs
   2. For Load Resources, as there is no energy component (demand), the energy and AS co-optimization process only allocates the offered MW capacity considering only AS offer(s)
5. If AS offers from Load Resources were cleared against the AS requirement from Load Resources, as there is no consideration of energy, then the corresponding AS MCPC will be determined solely by the marginal AS Offer from Load Resources – there is no opportunity cost for energy incorporated.

In the current ERCOT Nodal Market,

1. Load Resources are allowed to offer into any and all AS products.
2. AS Offers from Load Resources are cleared together with AS Offers from Generation Resources and thus,
3. The resultant MCPC reflects any opportunity costs for energy and other Ancillary Services. **This decision was there even in the Zonal Market for RRS and seems to reflect a policy decision made by the stakeholders.**
4. The equivalency ratio between MW offered for AS from Load Resources and MW offered for the same AS from Generation Resources is one (RAS = 1).

For the proposed future Ancillary Service product set, the same concepts (policy decisions) have been carried forth based on discussion at the FAST meetings,

1. PFR and FFR procurement is governed primarily by the constraint that incorporates the equivalency ratio R between PFR and FFR
   1. This is similar to the way RRS is currently cleared if we consider R=1
   2. Due to the equivalency ratio R, the MCPCRRS-LR for FFFR must reflect this ratio. Note: if in the current RRS, if there was a decision to incorporate the ratio R (which comes from reliability studies), then the MCPC for RRS from Load Resources must be R\*MCPCRRS-Gen
2. Similarly for CR (CR1&CR2) and SR (SR1&SR2), the CR and SR Offers from Generation Resource and Load Resource are cleared together to meet the total CR and SR requirement respectively. This is done to allow for substitution between the AS offers from Generation and Load Resources.

|  |  |  |
| --- | --- | --- |
| AS Product | MCPC | Comments |
| PFR |  | Shadow Price of the PFR+FFR1+FFR2 procurement constraint |
| FFR1 |  | R (equivalencing ratio of PFR to FFR) multiplied by the Shadow price of the PFR\_FFR1+FFR2 procurement constraint. This FFR2 MCPC is capped at VOLL |
| FFR2 |  | R (equivalencing ratio of PFR to FFR) multiplied by the Shadow price of the PFR+FFR1+FFR2 procurement constraint. This FFR1 MCPC is capped at VOLL |
| CR1 |  | Sum of the Shadow Prices of the CR (CR1+CR2) procurement constraint ()and the CR1 minimum procurement constraint () |
| CR2 |  | CR2 is valued the same as CR1. The Shadow Price if non- zero encapsulates any opportunity cost for energy and other Ancillary Service. |
| SR1 |  | Sum of the Shadow Prices of the SR (SR1+SR2) procurement constraint () and the SR1 minimum procurement constraint () |
| SR2 |  | SR2 is valued the same as SR1. The Shadow Price if non- zero encapsulates any opportunity cost for energy and other Ancillary Service. |
| RegUp |  | Shadow price of the RegUp procurement (including FRRS-Up) constraint |
| FRRS-Up |  | FRRS-Up is valued the same as RegUp |
| RegDn |  | Shadow price of the RegDn procurement (including FRRS-Dn) constraint |
| FRRS-Dn |  | FRRS-Down is valued the same as RegDn |