

Market Operations Bulletin #21 – December 21, 2004

Shadow Price Calculations for 5 Zone, 6 CSC model for 2005

Background

The purpose of this Market Bulletin is to summarize the Shadow Price calculations for 2005 for the 5 Zone, 6 CSC model.

Currently, ERCOT clears the Balancing Energy Market using a Linear Programming method. Following is the simplified model for this process. The software system that is used is much more complicated than the model presented below. The detail information is described in ESCA design document “ERCOT MOS Analytical Function Software Specification Document (SSD)”.

To minimize system cost, the following objective function is used to calculate Market Clearing price and Balancing Energy bid deployment.

Object:

$$\text{Min} \quad \sum_{i=1}^{NP} \sum_{j=1}^{NZ} BEUBP_{ij} * INSU_{ij} - \sum_{i=1}^{NP} \sum_{j=1}^{NZ} BEDBP_{ij} * INSD_{ij} \quad (1)$$

Subject to:

$$\sum_{i=1}^{NP} \sum_{j=1}^{NZ} P_{ij} - \sum_{j=1}^{NZ} LF_j + \sum_{i=1}^{NP} \sum_{j=1}^{NZ} (INSU_{ij} - INSD_{ij}) = 0 \quad (2)$$

$$\sum_{j=1}^{NZ} \left(\sum_{i=1}^{NP} (P_{ij} + INSU_{ij} - INSD_{ij}) - LF_j \right) * SF_{j,k} \leq TTC_k \quad k \in K_C \quad (3)$$

$$INSU_{ij} \leq MAXBEU_{ij} \quad i \in I_p, j \in J_Z \quad (4)$$

$$INSD_{ij} \leq MAXBED_{ij} \quad i \in I_p, j \in J_Z \quad (5)$$

$$\begin{aligned} (INSU_{ij,t} - INSD_{ij,t}) - (INSU_{ij,t-1} - INSD_{ij,t-1}) &\leq \min\left(\frac{\max(0, INSD_{ij,t-1} - INSU_{ij,t-1})}{Ramprate_{ij, Down}}, 10\right) * Ramprate_{ij, Down} \\ &+ [10 - \min\left(\frac{\max(0, INSD_{ij,t-1} - INSU_{ij,t-1})}{Ramprate_{ij, Down}}, 10\right)] * Ramprate_{ij, Up} \quad i \in I_p, j \in J_Z \end{aligned} \quad (6)$$

$$\begin{aligned} (INSU_{ij,t} - INSD_{ij,t}) - (INSU_{ij,t-1} - INSD_{ij,t-1}) &\geq -\min\left(\frac{\max(0, INSU_{ij,t-1} - INSD_{ij,t-1})}{Ramprate_{ij, Up}}, 10\right) * Ramprate_{ij, Up} \\ &- [10 - \min\left(\frac{\max(0, INSU_{ij,t-1} - INSD_{ij,t-1})}{Ramprate_{ij, Up}}, 10\right)] * Ramprate_{ij, Down} \quad i \in I_p, j \in J_Z \end{aligned} \quad (7)$$

In solving this LP, we can obtain shadow prices for all CSC Congestion constraints and shadow price for Power Balance. Then zonal MCPE can be calculated with the following equation:

$$MCPE_j = MCPE_{system} - \sum_k^{NC} SP_k * SF_{j,k} \quad k \in K \quad (8)$$

Re-write equation (8) using matrix:

$$\begin{bmatrix} MCPE_W \\ MCPE_N \\ MCPE_H \\ MCPE_S \\ MCPE_E \end{bmatrix} = \begin{bmatrix} 1 & -SF_{W,WN} & -SF_{W,SN} & -SF_{W,SH} & -SF_{W,NH} & -SF_{W,EN} & -SF_{W,NW} \\ 1 & -SF_{N,WN} & -SF_{N,SN} & -SF_{N,SH} & -SF_{N,NH} & -SF_{N,EN} & -SF_{N,NW} \\ 1 & -SF_{H,WN} & -SF_{H,SN} & -SF_{H,SH} & -SF_{H,NH} & -SF_{H,EN} & -SF_{H,NW} \\ 1 & -SF_{S,WN} & -SF_{S,SN} & -SF_{S,SH} & -SF_{S,NH} & -SF_{S,EN} & -SF_{S,NW} \\ 1 & -SF_{E,WN} & -SF_{E,SN} & -SF_{E,SH} & -SF_{E,NH} & -SF_{E,EN} & -SF_{E,NW} \end{bmatrix} \begin{bmatrix} MCPE_{system} \\ SP_{WN} \\ SP_{SN} \\ SP_{SH} \\ SP_{NH} \\ SP_{EN} \\ SP_{NW} \end{bmatrix} \quad (9)$$

If congestion does not occur on CSC WN, the Shadow Price of CSC WN is zero and the Shadow Prices of the other CSC can be verified by using Equation (10).

$$\begin{bmatrix} MCPE_{system} \\ SP_{SN} \\ SP_{SH} \\ SP_{NH} \\ SP_{EN} \\ SP_{NW} \end{bmatrix} = \begin{bmatrix} 1 & -SF_{W,SN} & -SF_{W,SH} & -SF_{W,NH} & -SF_{W,EN} & -SF_{W,NW} \\ 1 & -SF_{N,SN} & -SF_{N,SH} & -SF_{N,NH} & -SF_{N,EN} & -SF_{W,NW} \\ 1 & -SF_{H,SN} & -SF_{H,SH} & -SF_{H,NH} & -SF_{H,EN} & -SF_{H,NW} \\ 1 & -SF_{S,SN} & -SF_{S,SH} & -SF_{S,NH} & -SF_{S,EN} & -SF_{S,NW} \\ 1 & -SF_{E,SN} & -SF_{E,SH} & -SF_{E,NH} & -SF_{E,EN} & -SF_{E,NW} \end{bmatrix}^{-1} \begin{bmatrix} MCPE_W \\ MCPE_N \\ MCPE_H \\ MCPE_S \\ MCPE_E \end{bmatrix} \quad (10)$$

If congestion does not occur on CSC SN, the Shadow Price of CSC SN is zero and the Shadow Prices of the other CSC can be verified by using Equation (11).

$$\begin{bmatrix} MCPE_{system} \\ SP_{WN} \\ SP_{SH} \\ SP_{NH} \\ SP_{EN} \\ SP_{NW} \end{bmatrix} = \begin{bmatrix} 1 & -SF_{W,WN} & -SF_{W,SH} & -SF_{W,NH} & -SF_{W,EN} & -SF_{W,NW} \\ 1 & -SF_{N,WN} & -SF_{N,SH} & -SF_{N,NH} & -SF_{N,EN} & -SF_{N,NW} \\ 1 & -SF_{H,WN} & -SF_{H,SH} & -SF_{H,NH} & -SF_{H,EN} & -SF_{H,NW} \\ 1 & -SF_{S,WN} & -SF_{S,SH} & -SF_{S,NH} & -SF_{S,EN} & -SF_{S,NW} \\ 1 & -SF_{E,WN} & -SF_{E,SH} & -SF_{E,NH} & -SF_{E,EN} & -SF_{E,NW} \end{bmatrix}^{-1} \begin{bmatrix} MCPE_W \\ MCPE_N \\ MCPE_H \\ MCPE_S \\ MCPE_E \end{bmatrix} \quad (11)$$

If congestion does not occur on CSC SH, the Shadow Price of CSC SH is zero and the Shadow Prices of the other CSC can be verified by using Equation (12).

$$\begin{bmatrix} MCPE_{system} \\ SP_{WN} \\ SP_{SN} \\ SP_{NH} \\ SP_{EN} \\ SP_{NW} \end{bmatrix} = \begin{bmatrix} 1 & -SF_{W,WN} & -SF_{W,SN} & -SF_{W,NH} & -SF_{W,EN} & -SF_{W,NW} \\ 1 & -SF_{N,WN} & -SF_{N,SN} & -SF_{N,NH} & -SF_{N,EN} & -SF_{N,NW} \\ 1 & -SF_{H,WN} & -SF_{H,SN} & -SF_{H,NH} & -SF_{H,EN} & -SF_{H,NW} \\ 1 & -SF_{S,WN} & -SF_{S,SN} & -SF_{S,NH} & -SF_{S,EN} & -SF_{S,NW} \\ 1 & -SF_{E,WN} & -SF_{E,SN} & -SF_{E,NH} & -SF_{E,EN} & -SF_{E,NW} \end{bmatrix}^{-1} \begin{bmatrix} MCPE_W \\ MCPE_N \\ MCPE_H \\ MCPE_S \\ MCPE_E \end{bmatrix} \quad (12)$$

If congestion does not occur on CSC NH, the Shadow Price of CSC NH is zero and the Shadow Prices of the other CSC can be verified by using Equation (13).

$$\begin{bmatrix} MCPE_{system} \\ SP_{WN} \\ SP_{SN} \\ SP_{SH} \\ SP_{EN} \\ SP_{NW} \end{bmatrix} = \begin{bmatrix} 1 & -SF_{W,WN} & -SF_{W,SN} & -SF_{W,SH} & -SF_{W,EN} & -SF_{W,NW} \\ 1 & -SF_{N,WN} & -SF_{N,SN} & -SF_{N,SH} & -SF_{N,EN} & -SF_{N,NW} \\ 1 & -SF_{H,WN} & -SF_{H,SN} & -SF_{H,SH} & -SF_{H,EN} & -SF_{H,NW} \\ 1 & -SF_{S,WN} & -SF_{S,SN} & -SF_{S,SH} & -SF_{S,EN} & -SF_{S,NW} \\ 1 & -SF_{E,WN} & -SF_{E,SN} & -SF_{E,SH} & -SF_{E,EN} & -SF_{E,NW} \end{bmatrix}^{-1} \begin{bmatrix} MCPE_W \\ MCPE_N \\ MCPE_H \\ MCPE_S \\ MCPE_E \end{bmatrix} \quad (13)$$

If congestion does not occur on CSC EN, the Shadow Price of CSC EN is zero and the Shadow Prices of the other CSC can be verified by using Equation (14).

$$\begin{bmatrix} MCPE_{system} \\ SP_{WN} \\ SP_{SN} \\ SP_{SH} \\ SP_{NH} \\ SP_{NW} \end{bmatrix} = \begin{bmatrix} 1 & -SF_{W,WN} & -SF_{W,SN} & -SF_{W,SH} & -SF_{W,NH} & -SF_{W,NW} \\ 1 & -SF_{N,WN} & -SF_{N,SN} & -SF_{N,SH} & -SF_{N,NH} & -SF_{N,NW} \\ 1 & -SF_{H,WN} & -SF_{H,SN} & -SF_{H,SH} & -SF_{H,NH} & -SF_{H,NW} \\ 1 & -SF_{S,WN} & -SF_{S,SN} & -SF_{S,SH} & -SF_{S,NH} & -SF_{S,NW} \\ 1 & -SF_{E,WN} & -SF_{E,SN} & -SF_{E,SH} & -SF_{E,NH} & -SF_{E,NW} \end{bmatrix}^{-1} \begin{bmatrix} MCPE_W \\ MCPE_N \\ MCPE_H \\ MCPE_S \\ MCPE_E \end{bmatrix} \quad (14)$$

If congestion does not occur on CSC NW, the Shadow Price of CSC NW is zero and the Shadow Prices of the other CSC can be verified by using Equation (15).

$$\begin{bmatrix} MCPE_{system} \\ SP_{WN} \\ SP_{SN} \\ SP_{SH} \\ SP_{NH} \\ SP_{EN} \end{bmatrix} = \begin{bmatrix} 1 & -SF_{W,WN} & -SF_{W,SN} & -SF_{W,SH} & -SF_{W,NH} & -SF_{W,EN} \\ 1 & -SF_{N,WN} & -SF_{N,SN} & -SF_{N,SH} & -SF_{N,NH} & -SF_{N,EN} \\ 1 & -SF_{H,WN} & -SF_{H,SN} & -SF_{H,SH} & -SF_{H,NH} & -SF_{H,EN} \\ 1 & -SF_{S,WN} & -SF_{S,SN} & -SF_{S,SH} & -SF_{S,NH} & -SF_{S,EN} \\ 1 & -SF_{E,WN} & -SF_{E,SN} & -SF_{E,SH} & -SF_{E,NH} & -SF_{E,EN} \end{bmatrix}^{-1} \begin{bmatrix} MCPE_W \\ MCPE_N \\ MCPE_H \\ MCPE_S \\ MCPE_E \end{bmatrix} \quad (15)$$

where:

<i>i</i> :	Index of QSEs
<i>j</i> :	Index of zones
<i>t</i> :	Settlement interval
<i>I_p</i> :	A set of QSEs
<i>J_Z</i> :	A set of zones
<i>K_C</i> :	A set of Zonal congestions
<i>NC</i> :	Total number of CSCs
<i>NP</i> :	Total number of QSEs
<i>NZ</i> :	Total number of zones
<i>BEUBP_{ij}</i> :	Balancing Energy Up Bid price curve for QSE <i>i</i> in zone <i>j</i>
<i>BEDBP_{ij}</i> :	Balancing Energy Down Bid price curve for QSE <i>i</i> in zone <i>j</i>
<i>INSU_{ij}</i> :	Deployed amount of Balancing Energy Up Bid from QSE <i>i</i> in zone <i>j</i>
<i>INSD_{ij}</i> :	Deployed amount of Balancing Energy Down Bid from QSE <i>i</i> in zone <i>j</i>
<i>LF_j</i> :	Load Forecast in zone <i>j</i>
<i>MAXBEU_{ij}</i> :	Maximum amount of Balancing Energy Up bid from QSE <i>i</i> in zone <i>j</i>
<i>MAXBED_{ij}</i> :	Maximum amount of Balancing Energy Down bid from QSE <i>i</i> in zone <i>j</i>
<i>MCPE_j</i> :	Market Clearing Price in zone <i>j</i>
<i>MCPE_{system}</i> :	Power Balance Shadow Price
<i>P_{ij}</i> :	Total generation schedule of participant <i>i</i> in zone <i>j</i> , which includes both the balanced scheduled generation and mismatched scheduled generation
<i>Ramprate_{ij}</i> :	Portfolio Balancing Energy ramp rate submitted by the QSE <i>i</i> in zone <i>j</i>
<i>SF_{j,k}</i> :	Shift Factor of zone <i>j</i> to Zonal Constraint <i>k</i>
<i>SP_k</i> :	Shadow Price of CSC Congestion <i>k</i>
<i>TTC_k</i> :	Total Transfer Capacity of CSC Congestion Constraint <i>k</i>